Maxwell Equation (Differential Form)

First

I) $\nabla \cdot D = \rho$ (Gauss law of Electrostatics)

OR

$\nabla \cdot E = \rho/\epsilon_0$ (using $D = \epsilon_0 E$)

Second

II) $\nabla \cdot B = 0$ (Gauss law of Magnetostatics)

Third

III) $\nabla \times E = \frac{-dB}{dt}$ (Faraday law)

Fourth

IV) $\nabla \times B = \mu_0 \left(J + \frac{dD}{dt} \right)$ (Modified Ampere Circuital law)

OR

$\nabla \times H = \left(J + \frac{dD}{dt} \right)$ (using $B = \mu_0 H$)

Maxwell Equation (Integral Form)

First

I) $\oint E \cdot ds = \frac{q}{\epsilon_0}$

Second

II) $\oint B \cdot ds = 0$

Third

III) $\oint E \cdot dl = \frac{-d\Phi_B}{dt}$

Fourth

IV) $\oint B \cdot dl = \mu_0 \left(J + \frac{dD}{dt} \right) \cdot ds$

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where

D- Displacement or Electric Displacement
(Coulomb/M2)

ρ- Change Density (Coulom/M3)

B- Magnetic Induction (Wb/m2)
or Flux Density

H- Magnetic Field Intensity (Amp/M)

J- Current Density (I/A) (Current/Ampere)

First Maxwell Equation:-

$$\nabla \cdot E = \rho/\epsilon_0$$

Physical significance:-

→ It is based on gauss law of Electrostatics

→ Net Electric flux through a closed surface is equal to $\frac{1}{\epsilon_0}$ the total charge Enclosed by the surface.

$$\nabla \cdot E = \frac{q}{\epsilon_0} \qquad \oint_S E \cdot ds = \frac{1}{\epsilon_0} \int_V q \cdot dv$$

(Differential form) (Integral form)

→ It relate Electric flux with charge.

→ charge acts a a source or sink for the line of Electric force

Derivation:- Consider a surface bounded by a volume V in a medium having charge density as ρ.

then
$$q = \int_V \rho dV$$

using Gauss law of Electrostatics we have

$$\oint E \cdot ds = \frac{q}{\epsilon_0}$$

Subustituting $q = \int_V \rho dV$

then

$$\oint E \cdot ds = \frac{1}{\epsilon_0} \int \rho dV \quad -(2)$$

From Gauss theorm we have.

$$\oint_S E \cdot ds = \int_V (\nabla \cdot E)\, dv \quad -③$$

(Note Gauss Divergence theorm converts Surface Integral to volume Integral)

Equating ②, ③ we get.

$$\int_V (\nabla \cdot E)\, dv = \frac{1}{\epsilon_0} \int_V \rho\, dv$$

$$\int_V \left(\nabla \cdot E - \frac{\rho}{\epsilon_0} \right) dv = 0$$

$$\Rightarrow \boxed{\nabla \cdot E = \rho / \epsilon_0}$$

Maxwell ① Equation.

~~Maxwell's Second Equation~~ :-

Maxwell's Second Equation:-

$$\nabla \cdot B = 0$$

Significance:-

- It is based on Gauss law of Magnetostatics.
- It states that magnetic flux through any closed surface is zero

Differential form

$\nabla \cdot B = 0$

Integral form

$\oint B \cdot ds = 0$

- Time Independent Equation
- Magnetic flux is zero
- According to this Equation, isolated magnetic poles don't exist.

Derivation:- we have by Gauss law of Magnetostatics

$$\oint B \cdot ds = 0 \quad -(1)$$

By Gauss divergence theorm we have.

$$\oint_S B \cdot ds = \int_V (\nabla \cdot B)\, dv \quad -(2)$$

(Note:- By Gauss divergence theorm we have Surface Integral Equal to volume Integral)

Equating (1), (2)

$$\int_V (\nabla \cdot B)\, dv = 0$$

$$\boxed{\nabla \cdot B = 0} \rightarrow \text{Maxwell Second Equation}$$

Maxwell's third Equation:-

$$\nabla \times E = -\frac{dB}{dt}$$

Significance:-

⇒ This Equation represent Faraday law of Electro-Magnetic Induction.

Differential form | Integral form

$$\nabla \times E = -\frac{dB}{dt} \qquad \oint E \cdot dl = -\frac{d\phi}{dt}$$

→ It is time dependent Equation.

→ Relates space variation of E with variation of B

→ Time variation of Magnetic field generates Electric field.

→ Negative sign Justify Lenz law.

Derivation:- we have by Faraday law.

$$\oint_c E \cdot dl = -\frac{d\phi}{dt}$$

$$= -\frac{d(B \cdot ds)}{dt} \qquad (\phi = B \cdot ds) \quad -(1)$$

By Stokes theorm we have.

$$\oint_c E \cdot dl = \int_s (\nabla \times E) \cdot ds \quad -(2)$$

Equating (1) (2)

$$\oint_s (\nabla \times E)\, ds = -\frac{d}{dt}\int_s B\, ds$$

$$\oint_s (\nabla \times E)\, ds = \cancel{\frac{d}{dt}} - \int \frac{dB}{dt} \cdot ds$$

⇒ $\boxed{\nabla \times E = -\frac{dB}{dt}}$ — Maxwell Third Equn